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Derivatives and Their Applications



TEST AREAS

- finding derivatives of polynomial functions
- understanding and applying derivative rules
- determining the maximum and minimum of a function
- solving optimization and other application problems

Knowledge and Understanding

Circle the correct answers.

- ① Which of the following is the notation of a derivative for $y = f(x)$?
- A. $f'(x)$ B. y'
 C. $\frac{dy}{dx}$ D. all of the above
- ② Which is the definition of a derivative?
- A. $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$
 B. $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$
 C. $\lim_{x \rightarrow 0} \frac{f(x+h) - f(x)}{x+h}$
 D. A and B
- ③ If $f(x) = k$, where k is a constant, then
- A. $f'(x) = k$. B. $f'(x) = 1$.
 C. $f'(x) = 0$. D. $f'(x) = x$.
- ④ If $f(x) = x^n$, where n is a real number, then
- A. $f'(x) = nx$. B. $f'(x) = nx^{-1}$.
 C. $f'(x) = nx^n$. D. $f'(x) = nx^{n-1}$.
- ⑤ The constant multiple rule states that if $f(x) = kg(x)$, where k is a constant, then
- A. $f'(x) = kg(x)$. B. $f'(x) = kg'(x)$.
 C. $f'(x) = k + g(x)$. D. $f'(x) = g(x)g'(x)$.
- ⑥ Name the following rule: if $f(x) = p(x) - q(x)$, then $f'(x) = p'(x) - q'(x)$.
- A. the sum rule B. the difference rule
 C. the product rule D. the quotient rule
- ⑦ Which of the following is the product rule if $h(x) = f(x)g(x)$?
- A. $h'(x) = f'(x)g'(x)$
 B. $h'(x) = f'(x)g(x) + f(x)g'(x)$
 C. $h'(x) = f'(x)g(x) - f(x)g'(x)$
 D. $h'(x) = f'(x)g'(x) + f(x)g(x)$
- ⑧ Which of the following is the quotient rule if $h(x) = \frac{f(x)}{g(x)}$?
- A. $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$, $g(x) \neq 0$
 B. $h'(x) = \frac{f(x)g'(x) - f'(x)g(x)}{[g(x)]^2}$, $g(x) \neq 0$
 C. $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[f(x)]^2}$, $f(x) \neq 0$
 D. $h'(x) = \frac{f(x)g'(x) - f'(x)g(x)}{[f(x)]^2}$, $f(x) \neq 0$
- ⑨ If $h(x) = (f \circ g)(x)$, then
- A. $h'(x) = f'(g(x))$. B. $h'(x) = (f \circ g)(x)g'(x)$.
 C. $h'(x) = f'(g(x))g'(x)$. D. $h'(x) = f(x)g(x)g'(x)$.

⑩ Find the derivatives of each of the following functions.

a. $f(x) = 5x^2 + 2x$

b. $f(x) = 3x^3 - \frac{1}{2}x + 5$

c. $f(x) = x^6 + 7x^2 - 5x - 1$

d. $f(x) = 6\sqrt{x} + 3$

e. $f(x) = \frac{2}{x} - \frac{5}{x^2}$

f. $f(x) = 2\sqrt{x} - 6x + 7x^3$

g. $f(x) = x(x^2 + 1)$

h. $f(x) = (x + 3)(4x^4 - 2)$

i. $f(x) = \frac{1}{(x + 2)}$

j. $f(x) = \frac{(2x + 1)}{(x^2 - 1)}$

k. $f(x) = \frac{x^3}{(4 - x^2)}$

l. $f(x) = (x - 5)^2(x + 2)^3$

m. $f(x) = \sqrt{x^2 + 3x}$

n. $f(x) = 3(\sqrt{x} - 5)^2 + 1$

⑪ Determine the slope of the tangent to each of the curves at the given point.

a. $f(x) = \frac{x^3}{5 - 3x^2}$ (1, 0.5)

b. $f(x) = (x - 2)^2(x + 1)^3$ (-2, -16)



12 Complete the table with the derivatives.

$$f(x) = x^2$$

$$g(x) = 2x + 1$$

$$h(x) = \sqrt{x - 3}$$

	$y = f(x)h(x)$	$y = \frac{g(x)}{f(x)}$	$y = (h \circ g)(x)$
y'			
y''			

13 Find the point(s) where the tangent to the curve is horizontal.

a. $f(x) = -3x^2 + 108x - 722$

b. $f(x) = (x^2 + x + 1)(x - 1)$

14 Find the absolute maximum and minimum for each function with the given interval.

a. $f(x) = (x^2 + 3)(x + 3), -3 \leq x \leq 0$

b. $g(x) = \frac{x^2 + 4}{x}, 1 \leq x \leq 5$

Application

- ⑮ A ball falls from a cliff at a height of 425 m. The ball's height, h , in metres above the ground after t seconds can be modelled by the function $h(t) = 425 - 4.9t^2$. How fast is the ball falling at 4 seconds?
- ⑯ A 6500-L swimming pool is drained in 20 minutes. The volume of the water that remains in the pool after t minutes is modelled by $V(t) = 6500\left(1 - \frac{t}{20}\right)^2$, $0 \leq t \leq 20$. At what rate is the water flowing out of the pool when $t = 5$?
- ⑰ The position in metres of a car after t minutes can be modelled by $s(t) = (4 + 3t)^2\sqrt{t + 3}$. Determine the velocity of the car, $v(t)$. What is the velocity of the car at 6 minutes?
- ⑱ The volume of a balloon can be modelled by $V(r) = \frac{4}{3}\pi r^3$, where r is the radius in cm. Assume that the balloon is a perfect sphere. As the balloon is filled up, the radius is $r(t) = 0.5t^2$, $0 \leq t \leq 8$, where t is the time in seconds. At what rate will the volume of the balloon change with respect to time at 4 seconds?